# Map coloring via quantum simulation

Canadian map coloring was discussed in D-wave’s whitepaper [1]. This document tries to replicate the same with a much simpler maps due to limitation in number of qubits that can be simulated. Two maps – one simple (with five unique areas) and another map with five unique areas. Areas are equivalent to states in Canadian map problem.

## Simple map

I choose a map as below. Each area is numbered as indicated in the boxes

|  |  |  |
| --- | --- | --- |
|  | 1 |  |
| 2 | 3 | 4 |
|  | 5 |  |

The condition is that no two adjacent area will have same color.

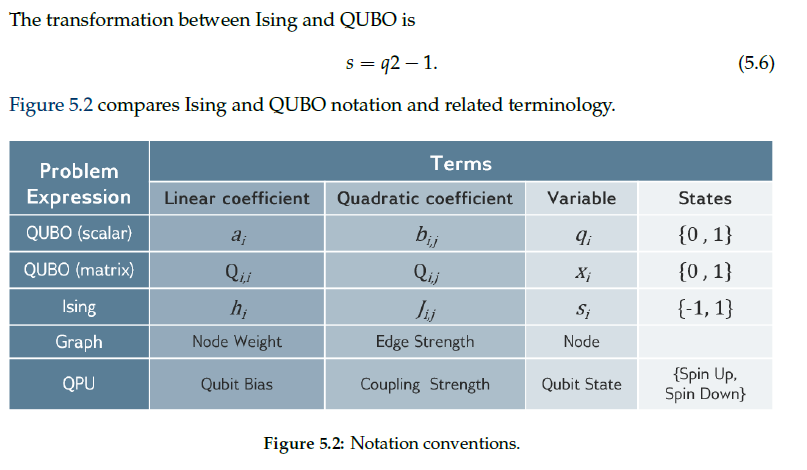
We will have two different colors indicated by two different qubits. Hence each area is indicated by two qubits.

First qubit down is for green and second qubit down is for blue.

The constraints are set-up so that two qubits for an area will neither is down or up simultaneously. The neighboring area will not have the green or blue indicating qubit down simultaneously.

Calculating the hs and Js Based on map-coloring whitepaper [1] on D-wave’s website. Please note that D-wave’s paper does the h and J calculation based on QUBO expression. This needs to be converted before it can be fed into my program for simulation.

Different notations are shown in getting started guide [2] I got from USRA manual section 5.2.3.



|  |  |  |
| --- | --- | --- |
| h and J for each area | |  |
| h | 0 | 0 |
| j | -0.5 |  |

For the complete map

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Map Area |  | 1 |  | 2 |  | 3 |  | 4 |  | 5 |  |
|  |  | blue | green | blue | green | blue | green | blue | green | blue | green |
| Qubit |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| hs |  | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

For Js. (all empty spaces are zeros)

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | 0 | -0.5 | 0 |  | -0.5 |  |  |  |  |  |
| 2 |  |  |  |  |  | -0.5 |  |  |  |  |
| 3 |  |  |  | -0.5 | -0.5 |  |  |  |  |  |
| 4 |  |  |  |  |  | -0.5 |  |  |  |  |
| 5 |  |  |  |  |  | -0.5 | -0.5 |  | -0.5 |  |
| 6 |  |  |  |  |  |  |  | -0.5 |  | -0.5 |
| 7 |  |  |  |  |  |  |  | -0.5 |  |  |
| 8 |  |  |  |  |  |  |  |  |  |  |
| 9 |  |  |  |  |  |  |  |  |  | -0.5 |
| 10 |  |  |  |  |  |  |  |  |  |  |

After running the simulation the results are displayed as

julia> qAnneal.qDisplay(b, 10, 5)

Math way of representing with probability amplitude.

(0.5407454f0 + 0.39953762f0im) |0101100101>

(0.5407454f0 + 0.39953762f0im) |1010011010>

(0.12553725f0 + 0.09339838f0im) |0101101111>

(0.12553725f0 + 0.09339838f0im) |1010010000>

(0.08471756f0 + 0.09561923f0im) |0101110100>

Physics way of representing with probabilities.

(0.4520359) |↓↑↓↑↑↓↓↑↓↑>

(0.4520359) |↑↓↑↓↓↑↑↓↑↓>

(0.024482857) |↓↓↓↓↑↓↓↑↓↑>

(0.024482857) |↑↑↑↑↓↑↑↓↑↓>

(0.016320102) |↑↑↓↑↓↓↓↑↓↑>

Top two options with highest probabilities are drawn as

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 1st solution |  |  |  |  |  |
| Map Area | 1 | 2 | 3 | 4 | 5 |
|  | green | green | blue | green | green |

|  |  |  |
| --- | --- | --- |
|  | 1 |  |
| 2 | 3 | 4 |
|  | 5 |  |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 2nd Solution |  |  |  |  |  |
| Map Area | 1 | 2 | 3 | 4 | 5 |
|  | blue | blue | green | blue | blue |

|  |  |  |
| --- | --- | --- |
|  | 1 |  |
| 2 | 3 | 4 |
|  | 5 |  |

Hence, both the high probability solutions provided correct answers.

## Another map

The map taken here is as below

|  |  |  |  |
| --- | --- | --- | --- |
| 1 | 2 | 3 | 5 |
| 4 | | |

This would require at least three colors to have adjacent areas colored differently. Three colors and hence three qubits to represent a color. First qubit down for blue, second qubit down for green, third qubit down for red.

|  |  |  |  |
| --- | --- | --- | --- |
| h and J for each area (scaled) | | |  |
| h | 1 | 1 | 1 |
| Js | -1 | -1 | -1 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Map Area |  | 1 |  |  | 2 |  |  | 3 |  |  | 4 |  |  | 5 |  |  |
|  |  | blue | green | red | blue | green | red | blue | green | red | blue | green | red | blue | green | red |
| Qubit |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| hs |  | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Js |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|  | Qubits | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|  | 1 | 1 | -1 | -1 | -0.5 |  |  |  |  |  | -1 |  |  |  |  |  |
|  | 2 |  | 1 | -1 |  | -0.5 |  |  |  |  |  | -1 |  |  |  |  |
|  | 3 |  |  | 1 |  |  | -0.5 |  |  |  |  |  | -1 |  |  |  |
|  | 4 |  |  |  | 1 | -1 | -1 | -0.5 |  |  | -1 |  |  |  |  |  |
|  | 5 |  |  |  |  | 1 | -1 |  | -0.5 |  |  | -1 |  |  |  |  |
|  | 6 |  |  |  |  |  | 1 |  |  | -0.5 |  |  | -1 |  |  |  |
|  | 7 |  |  |  |  |  |  | 1 | -1 | -1 | -1 |  |  | -1 |  |  |
|  | 8 |  |  |  |  |  |  |  | 1 | -1 |  | -1 |  |  | -1 |  |
|  | 9 |  |  |  |  |  |  |  |  | 1 |  |  | -1 |  |  | -1 |
|  | 10 |  |  |  |  |  |  |  |  |  | 1 | -1 | -1 | -1 |  |  |
|  | 11 |  |  |  |  |  |  |  |  |  |  | 1 | -1 |  | -1 |  |
|  | 12 |  |  |  |  |  |  |  |  |  |  |  | 1 |  |  | -1 |
|  | 13 |  |  |  |  |  |  |  |  |  |  |  |  | 1 | -1 | -1 |
|  | 14 |  |  |  |  |  |  |  |  |  |  |  |  |  | 1 | -1 |
|  | 15 |  |  |  |  |  |  |  |  |  |  |  |  |  |  | 1 |

The simulation results are

julia> qAnneal.qDisplay(b, 15, 10)

Math way of representing with probability amplitude.

(-0.43316334f0 - 0.25944436f0im) |111111111111111>

(-0.27115697f0 - 0.17783794f0im) |010001100010100>

(-0.27115616f0 - 0.17783602f0im) |001100010001010>

(-0.27115506f0 - 0.17783755f0im) |010100001010001>

(-0.27115545f0 - 0.17783685f0im) |001010100001100>

(-0.27115363f0 - 0.17783356f0im) |100001010100010>

(-0.27115262f0 - 0.17783482f0im) |100010001100001>

(-0.04166199f0 + 0.024884684f0im) |001110100010001>

(-0.04166203f0 + 0.024884434f0im) |010101100001010>

(-0.0416617f0 + 0.024884906f0im) |100011010001100>

Physics way of representing with probabilities.

(0.25494188) |↓↓↓↓↓↓↓↓↓↓↓↓↓↓↓>

(0.10515243) |↑↑↓↑↓↑↑↑↓↓↑↑↑↓↑>

(0.10515131) |↑↓↑↓↑↑↑↓↑↑↑↓↓↑↑>

(0.10515126) |↓↑↑↑↓↑↓↑↑↑↑↓↑↓↑>

(0.10515122) |↑↑↓↓↑↑↑↑↓↑↓↑↓↑↑>

(0.10514906) |↑↓↑↑↑↓↑↓↑↓↑↑↑↑↓>

(0.10514896) |↓↑↑↑↑↓↓↑↑↑↓↑↑↑↓>

(0.0023549688) |↓↑↑↑↓↑↑↑↓↑↓↓↓↑↑>

(0.00235496) |↑↓↑↓↑↑↑↑↓↓↑↓↑↓↑>

(0.0023549558) |↑↑↓↓↑↑↑↓↑↓↓↑↑↑↓>

The first qubit arrangement is not valid as only one qubit in an area can be down. However, the next six qubits with same probability seem valid choices and are deciphered as below

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| area -> | 1 | 2 | 3 | 4 | 5 |
| Solution 1 | red | green | red | blue | green |
| Solution 2 | green | blue | green | red | blue |
| Solution 3 | blue | green | blue | red | green |
| Solution 4 | red | blue | red | green | blue |
| Solution 5 | green | red | green | blue | red |
| Solution 6 | blue | red | blue | green | red |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| 1 | 1 | 2 | 3 | 5 |
|  | 4 | | |
|  |  |  |  |  |
|  |  |  |  |  |
| 2 | 1 | 2 | 3 | 5 |
|  | 4 | | |
|  |  |  |  |  |
|  |  |  |  |  |
| 3 | 1 | 2 | 3 | 5 |
|  | 4 | | |
|  |  |  |  |  |
|  |  |  |  |  |
| 4 | 1 | 2 | 3 | 5 |
|  | 4 | | |
|  |  |  |  |  |
|  |  |  |  |  |
| 5 | 1 | 2 | 3 | 5 |
|  | 4 | | |
|  |  |  |  |  |
|  |  |  |  |  |
| 6 | 1 | 2 | 3 | 5 |
|  | 4 | | |

Hence, all these six choices are valid choices.

## References

[1] Programming with D-wave:Map coloring problem, Dhal, E. D., November 2013.

[2] Getting Started with D-wave System, User Manual,2019-02-19